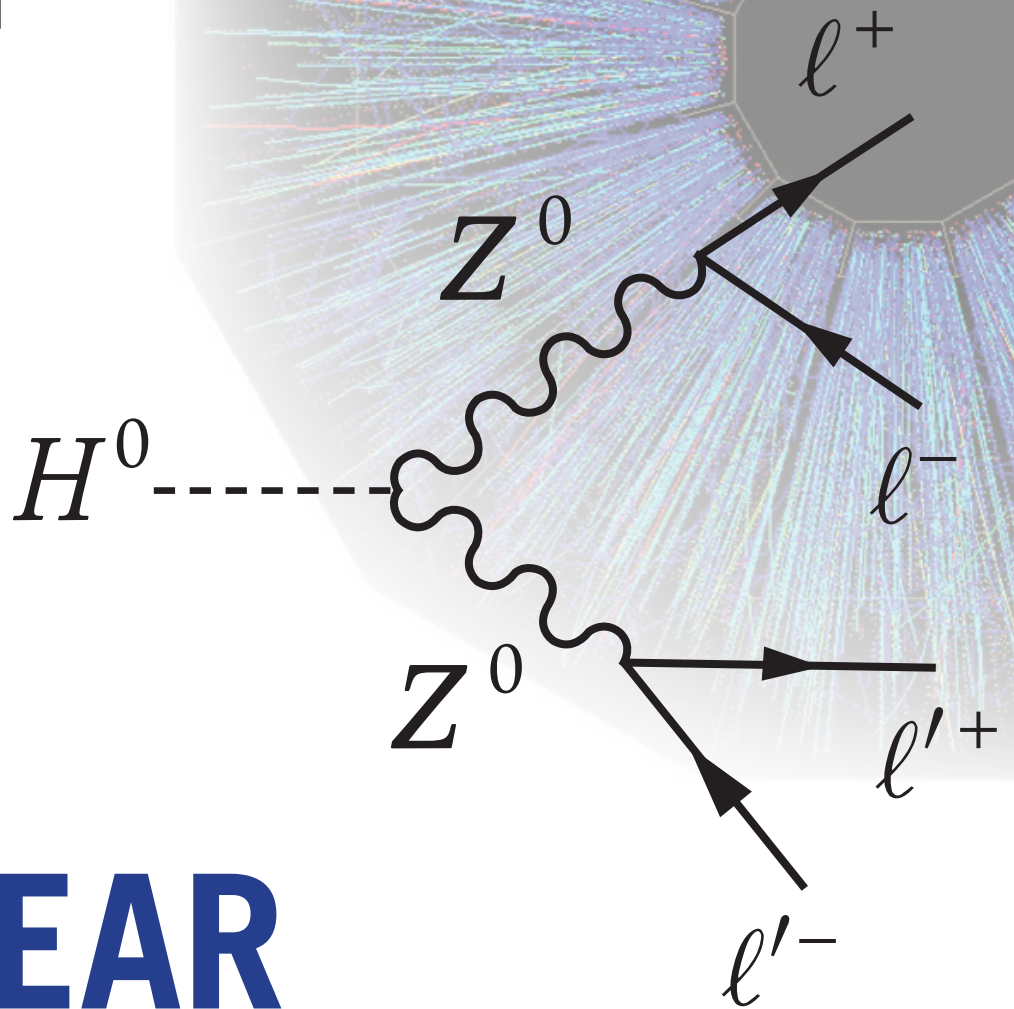


BRIAN R. MARTIN
GRAHAM SHAW



3RD EDITION

NUCLEAR AND PARTICLE PHYSICS

AN INTRODUCTION



WILEY

Nuclear and Particle Physics

Nuclear and Particle Physics

An Introduction

Third Edition

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Preface

It is common practice to teach nuclear physics and particle physics together in an introductory undergraduate course, and it is for such a course that this book has been written. The material is presented so that different selections can be made for a short course of about 25–30 lectures depending on the lecturer's preferences and the students' backgrounds. On the latter, students should have taken a first course in quantum physics, covering the traditional topics in nonrelativistic quantum mechanics and atomic physics. No prior knowledge of nuclear and particle physics is assumed. A few lectures on relativistic kinematics would also be useful, but this is not essential, as the necessary background is given in an appendix and is only used in a few places in the book.

We have not presented proofs or derivations of all the statements in the text. Rather, we have taken the view that it is more important that students see an overview of the subject, which for many, probably the majority, will be the only time they study nuclear and particle physics. For future specialists, the details will form part of more advanced courses. We have tried to take a direct approach throughout, focusing on the interpretation of experimental data in terms of current models and theories. Space restrictions have still meant that it has been necessary to make a choice of topics, and doubtless other equally valid choices could have been made. This is particularly true in Chapter 9, which deals with applications of nuclear and particle physics.

Since publication of the Second Edition of this book, there have been many important developments in both nuclear and particle physics. These include: the long-awaited discovery of the Higgs boson; substantial progress in neutrino physics and symmetry breaking in the weak interaction; a better understanding of stellar evolution and cosmology; high-precision nuclear mass measurements; increased developments in applying nuclear and particle physics techniques to clinical science; and tighter constraints on difficult-to-measure quantities, such as possible electric dipole moments and the masses of hypothetical particles, which are important for testing new theories of particle physics. Our aim in producing this Third Edition is again to bring the book up-to-date throughout, while leaving its basic philosophy unchanged. In doing this we are grateful to John Wiley and Sons for permission to use material from other books that we have published with them.

Finally, a word about footnotes: readers often have strong views about these ('Notes are often necessary, but they are necessary evils' – Samuel Johnson), so, as in previous editions, in this book they are designed to provide 'non-essential' information only. For those readers who prefer not to have the flow disrupted, ignoring the footnotes should not detract from understanding the text. Nuclear and particle physics have been, and still are, very important parts of the entire subject of physics and its practitioners have won an impressive number of Nobel Prizes. For historical interest, the footnotes also record many of these awards.

Brian Martin and Graham Shaw
July 2018

Notes

References

References are referred to in the text in the form of a name and date, for example Jones (1997). A list of references with full publication details is given at the end of the book.

Data

Tabulations of nuclear and particle physics data, such as masses, quantum numbers, decay modes, etc., are now readily available at the ‘click of a mouse’ from a number of sites and it is useful for students to get some familiarity with such sources. They are also needed to solve some end-of-chapter problems in the book. Many physical quantities are also readily found by a simple Internet search.

For particle physics, a comprehensive compilation of data, plus brief critical reviews of a number of current topics, may be found in the biannual publications of the Particle Data Group (PDG). The 2018 edition of their definitive *Review of Particle Properties* is referred to in Tanabashi et al. (Particle Data Group) (2018). *Physical Review D*98, 030001 in the references, and also as Particle Data Group (2018). The PDG Review is available online at <http://pdg.lbl.gov> and this site also contains links to other sites where compilations of specific particle data may be found.

Nuclear physics does not have the equivalent of the PDG review, but extensive compilations of nuclear data are available from a number of sources. Examples are: the Berkeley Laboratory Isotopes Project (<http://ie.lbl.gov/education/isotopes.htm>); the National Nuclear Data Center (NNDC), based at Brookhaven National Laboratory, USA (<http://www.nndc.bnl.gov>); the Nuclear Data Centre of the Japan Atomic Energy Research Institute (<http://wwwndc.tokai-sc.jaea.go.jp/NuC>); and the Nuclear Data Evaluation Laboratory of the Korea Atomic Energy Research Institute (<http://atom.kaeri.re.kr>). All four sites have links to other data compilations.

Problems

Problems are provided for Chapters 1 to 9 and Appendices A to D; they are an integral part of the text. The problems are sometimes numerical and require values of physical constants that are given on the inside rear cover. Some also require data that may be found in the reference sites listed above. Short answers to selected problems are given at the end of the book in Appendix E. Readers may access the full solutions to odd-numbered problems on the book’s website given below, and instructors can access there the full solutions for all problems.

Illustrations

Some illustrations in the text have been adapted from, or are based on, diagrams that have been published elsewhere. We acknowledge, with thanks, permission to use such illustrations from the relevant copyright holders, as stated in the captions.

Website

www.wiley.com/go/martin/nuclear3

Instructors may access PowerPoint slides of all the illustrations from the text on the accompanying website. As indicated above, solutions for all the problems are also available to Instructors, with odd-numbered solutions available to all Readers. Any misprints or other necessary corrections brought to the author's notice will be listed. We would also be grateful for any other comments about the book, which should initially be sent to the Publishers (jcossham@wiley.com).

Basic concepts

1.1 History

Although this book will not follow a strictly historical development, to ‘set the scene’ this first chapter will start with a brief review of the most important discoveries that led to the separation of nuclear physics from atomic physics as a subject in its own right, and later work that in its turn led to the emergence of particle physics from nuclear physics.¹

1.1.1 The origins of nuclear physics

In 1896 Becquerel observed that photographic plates were being fogged by an unknown radiation emanating from uranium ores. He had accidentally discovered *radioactivity*, the fact that some chemical elements spontaneously emit radiation. The name was coined by Marie Curie two years later to distinguish this phenomenon from induced forms of radiation. In the years that followed, radioactivity was extensively investigated, notably by the husband and wife team of Pierre and Marie Curie, and by Rutherford and his collaborators.² Other radioactive sources were quickly found, including the hitherto unknown chemical elements polonium and radium,

¹For a readable and lavishly illustrated account, see Close, Marten, and Sutton (1987). An interesting account of the early period, with descriptions of the personalities involved, is given in Segrè (1980), while a very detailed and scholarly account may be found in Pais (1986).

²The 1903 Nobel Prize in Physics was awarded jointly to Henri Becquerel for his discovery and to Pierre and Marie Curie for their subsequent research into radioactivity. Ernest Rutherford had to wait until 1908, when he was awarded the Nobel Prize in Chemistry for his ‘investigations into the disintegration of the elements and the chemistry of radioactive substances’.

discovered by the Curies in 1897.³ It was soon established that there were two distinct types of radiation involved, named by Rutherford α and β rays. We know now that β rays are electrons (the name ‘electron’ had been coined in 1894 by Stoney) and α rays are doubly ionised helium atoms. In 1900 a third type of decay was discovered by Villard that involved the emission of photons, the quanta of electromagnetic radiation, referred to in this context as γ rays. These historical names are still commonly used.

The revolutionary implications of these experimental discoveries did not become fully apparent until 1902. Prior to this, atoms were still believed to be immutable – indestructible and unchanging – an idea with its origin in Greek philosophy and, for example, embodied in Dalton’s atomic theory of chemistry in 1804. This causes a big problem: if the atoms in a radioactive source remain unchanged, where does the energy carried away by the radiation come from? Typically, early attempts to explain the phenomena of radioactivity assumed that the energy was absorbed from the atmosphere or, when that failed, that energy conservation was violated in radioactive processes. However, Rutherford had shown in 1900 that the intensity of the radiation emitted from a radioactive source was not constant, but reduced by a factor of two in a fixed time that was characteristic of the source, but independent of its amount. This is called its *half-life*. In 1902, together with Soddy, he put forward the correct explanation, called the *transformation theory*, according to which the atoms of any radioactive element decay with a characteristic half-life, emitting radiation, and in so doing are transformed into the atoms of a different chemical element. The centuries old belief in the immutability of atoms was shattered forever.

An important question not answered by the transformation theory is: which elements are radioactive and which are stable? An early attempt to solve this problem was made by J.J. Thomson, who was extending the work of Perrin and others on the radiation that had been observed to occur when an electric field was established between electrodes in an evacuated glass tube. In 1897 he was the first to definitively establish the nature of these ‘cathode rays’. We now know they consist of free electrons, denoted e^- (the superscript denotes the electric charge) and Thomson measured their mass and charge.⁴ This gave rise to the speculation that atoms contained electrons in some way, and in 1903 Thomson suggested a model where the electrons were embedded and free to move in a region of positive charge filling the entire volume of the atom – the so-called *plum pudding model*. This model could account for the stability of atoms, but gave no explanation for the discrete wavelengths observed in the spectra of light emitted from excited atoms.

³For these discoveries, Marie Curie won a second Nobel Prize in 1911, this time in Chemistry. The honour would presumably have been shared with her husband had he not been killed in a road accident in 1906.

⁴J.J. Thomson received the 1906 Nobel Prize in Physics for his discovery. A year earlier, Philipp von Lenard had received the 1905 Physics Prize for his work on cathode rays.

The plum pudding model was finally ruled out by a classic series of experiments suggested by Rutherford and carried out by his collaborators Geiger and Marsden starting in 1909. This consisted of scattering α particles from very thin gold foils. In the Thomson model, most of the α particles would pass through the foil, with only a few suffering deflections through small angles. However, Geiger and Marsden found that some particles were scattered through very large angles, even greater than 90° . As Rutherford later recalled, ‘It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you’.⁵ He then showed that this behaviour was not due to multiple small-angle deflections, but could only be the result of the α particles encountering a very small, very heavy, positively charged central *nucleus*. (The reason for these two different behaviours is discussed in Appendix C.)

To explain these results, Rutherford in 1911 proposed the *nuclear model* of the atom. In this model, the atom was likened to a planetary system, with the light electrons (the ‘planets’) orbiting about a tiny but heavy central positively charged nucleus (the ‘sun’). The size of the atom is thus determined by the radii of the electrons’ orbits, with the mass of the atom arising almost entirely from the mass of the nucleus. In the simplest case of hydrogen, a single electron orbits a nucleus, now called the *proton* (p), with electric charge $+e$, where e is the magnitude of the charge on the electron, to ensure that hydrogen atoms are electrically neutral. Alpha particles are just the nuclei of helium, while heavier atoms were considered to have more electrons orbiting heavier nuclei. At about the same time, Soddy showed that a given chemical element often contained atoms with different atomic masses but identical chemical properties. He called this *isotopism* and the members of such families *isotopes*. Their discovery led to a revival of interest in *Prout’s Law* of 1815, which claimed that all the elements had integer atomic mass in units of the mass of the hydrogen atom, called *atomic weights*. This holds to a good approximation for many elements, like carbon and nitrogen, with atomic weights of approximately 12.0 and 14.0 in these units, but does not hold for other elements, like chlorine, which has an atomic weight of approximately 35.5. However, such fractional values could be explained if the naturally occurring elements consisted of mixtures of isotopes. Chlorine, for example, is now known to consist of a mixture of isotopes with atomic weights of approximately 35.0 and 37.0, giving an average value of 35.5 overall.⁶

Although the planetary model explained the α particle scattering experiments, there remained the problem of reconciling it with the observation of stable atoms. In classical physics, the electrons in the planetary model would be continuously accelerating and would therefore lose energy by radiation, leading to the collapse of the atom. This problem was solved by Bohr in 1913, who revolutionised the study of atomic physics by

⁵Quoted on p. 111 of da C. Andrade (1964).

⁶Frederick Soddy was awarded the 1921 Nobel Prize in Chemistry for his work on isotopes.

applying the newly emerging quantum theory. The result was the Bohr–Rutherford model of the atom, in which the motion of the electrons is confined to a set of discrete orbits. Because photons of a definite energy would be emitted when electrons moved from one orbit to another, this model could explain the discrete nature of the observed electromagnetic spectra when excited atoms decayed. In the same year, Moseley extended these ideas to a study of X-ray spectra and conclusively demonstrated that the charge on the nucleus is $+Ze$, where the integer Z was the atomic number of the element concerned, and implying Z orbiting electrons for electrical neutrality. In this way he laid the foundation of a physical explanation of Mendeleev’s periodic table and in the process predicted the existence of no less than seven unknown chemical elements, which were all later discovered.⁷

The phenomena of atomic physics are controlled by the behaviour of the orbiting electrons and are explained in detail by refined modern versions of the Bohr–Rutherford model, including relativistic effects described by the Dirac equation, the relativistic analogue of the Schrödinger equation that applies to electrons, which is discussed in Section 1.2. However, following the work of Bohr and Moseley it was quickly realised that radioactivity was a nuclear phenomenon. In the Bohr–Rutherford and later models, different isotopes of a given element have different nuclei with different nuclear masses, but their orbiting electrons have virtually identical chemical properties because these nuclei all carry the same charge $+Ze$. The fact that such isotopes often have dramatically different radioactive decay properties is therefore a clear indication that these decays are nuclear in origin. In addition, since electrons were emitted in β decays, it seemed natural to assume that nuclei contained electrons as well as protons, and the first model of nuclear structure, which emerged in 1914, assumed that the nucleus of an isotope of an element with atomic number Z and mass number A was itself a tightly bound compound of A protons and $A - Z$ electrons. This provided an explanation of the existence of isotopes and of the approximate validity of Prout’s law when applied to isotopes, because the electron mass is negligible compared to that of the proton. However, although this model persisted for some time, it was subsequently ruled out by detailed measurements of the spins of nuclei (cf. Problem 1.1).

The correct explanation of isotopes and nuclear structure had to wait almost twenty years, until a classic discovery by Chadwick, in 1932. His work followed earlier experiments by Irène Curie (the daughter of Pierre and Marie Curie) and her husband Frédéric Joliot. They had observed that neutral radiation was emitted when α particles bombarded beryllium, and later work had studied the energy of protons emitted when paraffin

⁷Niels Bohr received the 1922 Nobel Prize in Physics for his theoretical work on the structure of atoms. Moseley was nominated for the 1915 Nobel Prizes in both Physics and Chemistry for his pioneering use of X-rays, but was tragically killed in World War I in August 1915 at the age of 27, before a decision was made.

was exposed to this neutral radiation. Chadwick refined and extended these experiments and demonstrated that they implied the existence of an electrically neutral particle of approximately the same mass as the proton, called the *neutron* (n).⁸ The discovery of the neutron led immediately to the correct formulation of nuclear structure, in which an isotope of atomic number Z and mass number A is a bound state of Z protons and $A - Z$ neutrons. There are no electrons bound inside nuclei.

Finally, to complete this historical account, we must go back to another major result: the discovery of the continuous β -decay spectrum by Chadwick in 1914. At that time, nuclear decays were all viewed as a parent nucleus decaying via α , β , or γ decay to give a daughter nucleus plus either an alpha particle, an electron or a photon, respectively. As each possibility would be a two-body decay, energy and momentum conservation implies that the emitted particle would have a unique energy, depending on the masses of the parent and daughter nucleons, which would be the same for all observed decays of a given type. This behaviour is precisely what is observed for α decays and γ decays and the earliest experiments erroneously suggested the same held for β decays. However, when Chadwick measured the energies of the electrons from samples of nuclei he found that the electrons emitted in a given β -decay process had a continuous energy distribution, as shown in Figure 1.1.

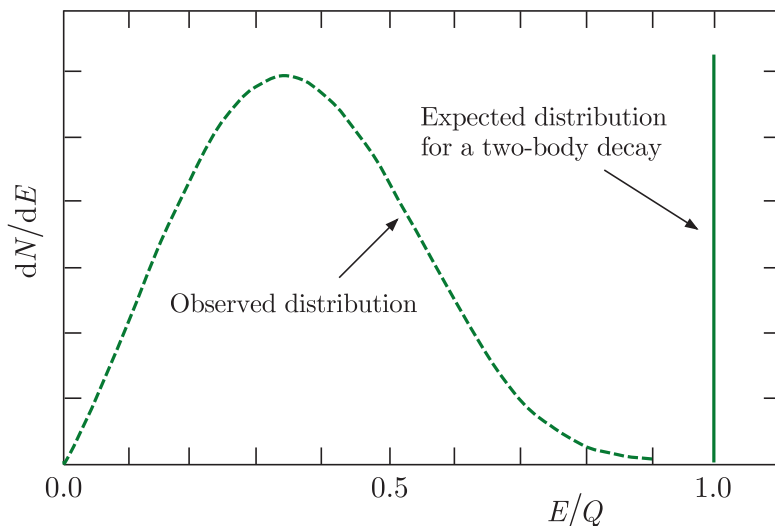


Figure 1.1 The observed electron energy distribution dN/dE in β decay (dashed line) as a function of E/Q , where E is the kinetic energy of the electron and Q is the total energy released. Also shown is the expected energy distribution if β decay were a two-body process (solid line).

After a hiatus due to the first world war, various ideas were suggested to explain this unexpected result, including a remarkable proposal by Bohr in 1929 that energy conservation was violated in β decays, but later abandoned by him in favour of the correct hypothesis proposed by Pauli in

⁸James Chadwick received the 1935 Nobel Prize in Physics for his discovery of the neutron. The discovery was not unexpected, because Rutherford had already deduced that the nucleus must include uncharged constituents with masses similar to that of the proton, and had even coined the name ‘neutron’. Irène Curie and Frédéric Joliot received the 1935 Nobel Prize in Chemistry for ‘synthesising new radioactive elements’.

1930. Pauli proposed that an additional, and hitherto unknown, neutral particle was emitted in β decays and shared the energy released with the electron. This particle had to be very light, since the most energetic electrons in the observed continuous distribution carried off almost all the energy released in the decay, as can be seen in Figure 1.1; it had also to interact so weakly with matter that it invariably escaped detection. Despite this, its existence was rapidly accepted, largely because of its crucial role in the highly successful theory of β decay proposed in 1932 by Fermi, who used the name *neutrino* (meaning ‘little neutral one’) for the new particle after his close friend and colleague Amaldi jokingly suggested it to distinguish Pauli’s particle from Chadwick’s ‘big neutral one’, the neutron.⁹

In conclusion, by 1932 physicists had arrived at a model of the nucleus in which an isotope of atomic number Z and mass number A is a bound state of Z protons and $A - Z$ neutrons. Later workers, including Heisenberg, another of the founders of quantum theory, applied quantum mechanics to the nucleus, now viewed as a collection of neutrons and protons, collectively called *nucleons*. In this case, however, the force binding the nucleus is not the electromagnetic force that holds electrons in their orbits, but a much stronger force that does not depend on the charge of the nucleon (i.e. is charge-independent) and with a very short effective range. This binding interaction is called the *strong nuclear force*. In addition, there is a third force, much weaker than the electromagnetic force, called the *weak interaction*, responsible for β decays, where neutrinos as well as electrons are emitted. These ideas form the essential framework of our understanding of the nucleus today. Nevertheless, there is still no single theory that is capable of explaining all the data of nuclear physics and we shall see that different models are used to interpret different classes of phenomena.

1.1.2 The emergence of particle physics: hadrons and quarks

By the early 1930s, the nineteenth century view of atoms as indivisible elementary particles had been replaced and a smaller group of subatomic particles now enjoyed this status: electrons, protons and neutrons. To these we must add two electrically neutral particles: the photon (γ) and the neutrino (ν). However, this simple picture was not to last, because of the discovery of many new subatomic particles, initially in cosmic rays and later in experiments using particle accelerators.

We start with cosmic rays, which may be conveniently divided into two types: *primaries*, which are high-energy particles, mostly protons,

⁹The neutrino was eventually detected, but not until very much later, by Reines and Cowan in 1956. A brief description of their experiment is given in Section 2.1.1 of Martin and Shaw (2017) and in more detail in Chapter 12 of Trigg (1975). Frederick Reines shared the 1995 Nobel Prize in Physics for his work in neutrino physics, particularly for the detection of the neutrino.

incident on the Earth's atmosphere from all directions in space; and *secondaries*, which are produced when the primaries collide with nuclei in the Earth's atmosphere, with some penetrating to sea level. It was among these secondaries that the new particles were discovered, mainly using a detector devised by C.T.R. Wilson, called the *cloud chamber*. It consisted of a vessel filled with air almost saturated with water vapour and fitted with an expansion piston. When the vessel was suddenly expanded, the air was cooled and became supersaturated. Droplets were then formed preferentially along the trails of ions left by charged particles passing through the chamber. Immediately after the expansion, the chamber was illuminated by a flash of light and the tracks of droplets so revealed were photographed before they had time to disperse. The use of these chambers in cosmic ray studies led to many important discoveries, including, in 1932, the detection of *antiparticles*, to be discussed in Section 1.2.¹⁰ However, the birth of particle physics as a new subject, distinct from atomic and nuclear physics, dates from 1947 with the discovery of *pions* and of *strange particles* by cosmic ray groups at Bristol and Manchester Universities, respectively. We will consider these in turn.

The discovery of pions was not unexpected, since Yukawa had famously predicted their existence in a theory of the strong nuclear forces proposed in 1934. We will return to this in Section 1.5. Here we will simply note that the range of the nuclear force required the pions to have a mass of around one seventh of the proton mass, while the charge independence of the nuclear force required there to be three charge states, denoted π^+ , π^- , π^0 , with charges $+e$, $-e$ and zero, respectively. This gave rise to a search for these particles in cosmic ray secondaries, and in 1936 Anderson and Neddermeier discovered new subatomic particles that were initially thought to be pions, but are now known to be particles called *muons*. As we shall see in Chapter 3, muons are rather like heavy electrons and, like both electrons and neutrinos, do not interact via the strong force that holds the nucleus together. Charged pions with suitable properties were finally detected in 1947 using photographic emulsions containing a silver halide. The ionisation energy deposited by a charged particle passing through the emulsion causes the formation of a latent image, and the silver grains resulting from subsequent development form a visual record of the path of the particle. The neutral pion was detected somewhat later in 1950.¹¹ Pions interact with each other and with nucleons via forces comparable in strength to the strong nuclear interaction between nucleons and in future we will refer to all such forces as *strong interactions*, reserving the term

¹⁰Wilson built the first cloud chamber in 1911 and shared the 1927 Nobel Prize in Physics. Victor Hess discovered cosmic rays in 1912, by making a series of balloon flights and showing that the intensity of radiation increased at high altitudes, indicating an extraterrestrial origin. He shared the 1936 Nobel Prize in Physics.

¹¹The 1949 Nobel Prize in Physics was awarded to Hideki Yukawa for his prediction of the pion and in 1950 the Nobel Prize in Physics was awarded to Cecil Powell for his leading role in its discovery.

strong nuclear interaction to the special case of nucleon–nucleon interactions. Particles that interact by the strong force are now called *hadrons*. Thus pions and nucleons are examples of hadrons, while electrons, muons and neutrinos are not.

Further work using cloud chambers to detect cosmic ray secondaries led to the discovery in 1947 by Rochester and Butler of new particles, named *kaons*, which, in contrast to the discovery of pions, was totally unexpected. Kaons were almost immediately recognised as a completely new form of matter, because they had supposedly ‘strange’ properties, which will be discussed further in Section 3.3. Other *strange particles* with similar properties were discovered, and in 1953 it was realised that these properties were precisely what would be expected if they were hadrons with nonzero values of an hitherto unknown quantum number, given the name *strangeness* by Gell-Mann, which was conserved in strong and electromagnetic interactions, but not necessarily conserved in the so-called *weak interactions* responsible for β decay. Non-strange particles like the pions and nucleons have zero values of strangeness. This led Gell-Mann, and independently Zweig, to suggest that hadrons were composed of more fundamental particles called *quarks* (q), together with their antiparticles. Three quarks were required at the time, denoted u , d , and s , with fractional electric charges $+2e/3$, $-e/3$, and $-e/3$, respectively. In particular, ordinary matter, i.e. protons and nucleons are composed of u and d quarks only, while the strange particles also contain s quarks. The latter is called the *strange quark* and the strangeness quantum number merely reflects the number of strange quarks and/or antiquarks present.

The 1950s also saw technological developments that enabled high-energy beams of particles to be produced in laboratories, and these rapidly replaced cosmic rays as the source of the high-energy particles required to create new particles in collisions. At the same time, cloud chambers were largely superseded by bubble chambers, a more efficient device in which charged particles were detected by the trail of bubbles left along their tracks through a superheated liquid, rather than droplets in a supercooled gas.¹² By the mid-1960s this had resulted in the discovery of many more unstable particles and the above *quark model* had considerable success in understanding the properties of the observed hadrons, as we shall see in Section 3.3,¹³ but because no free quarks were detected experimentally, there was initially considerable scepticism for this interpretation. We now know that there is a fundamental reason why quarks cannot

¹²Many beautiful pictures of events observed in both cloud and bubble chambers may be found in Close Marten, and Sutton (1987). Donald Glaser was awarded the 1960 Nobel Prize in Physics for his invention of the bubble chamber and Luis Alvarez received the 1968 prize for its further development and use in discovering new subatomic particles.

¹³Murray Gell-Mann received the 1969 Nobel Prize in Physics for ‘contributions and discoveries concerning the classification of elementary particles and their interactions’. For the origin of the word ‘quark’, he cited the now famous quotation ‘Three quarks for Muster Mark’ from James Joyce’s book *Finnegans Wake*. George Zweig had suggested the name ‘aces’. Subsequently, more than three quarks were discovered, as we shall see.

be observed as free particles (it is discussed in Section 5.1), but at the time many physicists looked upon quarks as a convenient mathematical description, rather than physical particles. However, evidence for the existence of quarks as real particles began to emerge in 1969 from a series of experiments analogous to those of Rutherford and his co-workers, where high-energy beams of electrons and neutrinos were scattered from nucleons. (These experiments are discussed in Section 5.5.) Analysis of the angular distributions of the scattered particles confirmed that the nucleons were themselves bound states of point-like charged entities, with properties consistent with those hypothesised in the quark model, including their fractional electric charges. This is essentially the picture today, where elementary particles are considered to be a small number of fundamental physical entities, including quarks, the electron, neutrinos, the photon and a few others we shall meet, but no longer nucleons.

1.1.3 The standard model of particle physics

Following the discovery of quarks, an ‘in principle’ complete theory of elementary particles gradually emerged, called, rather prosaically, the *standard model*. This aims to explain all the phenomena of particle physics, except those due to gravity, in terms of the properties and interactions of a small number of *elementary* (or *fundamental*) *particles*, which are now defined as being point-like, without internal structure or excited states. Particle physics thus differs from nuclear physics in having a single theory to interpret its data. Here we restrict ourselves to a brief outline of the standard model, which will be developed in more detail later in Chapters 3, 5, 6, and 7.

An elementary particle is characterised by, amongst other things, its mass, its electric charge and its *spin*. The latter is a permanent angular momentum possessed by all particles in quantum theory, even when they are at rest. Spin has no classical analogue and is not to be confused with the use of the same word in classical physics, where it usually refers to the angular momentum of extended objects. The maximum value of the spin angular momentum about any axis is $S\hbar$ ($\hbar \equiv h/2\pi$), where h is Planck’s constant and S is the *spin quantum number*, or *spin* for short. It has a fixed value for particles of any given type (for example $S = 1/2$ for electrons) and general quantum mechanical principles restrict the possible values of S to be 0, $1/2$, 1, $3/2$, \dots . Particles with half-integer spin are called *fermions* and those with integer spin are called *bosons*. There are two families of elementary fermions in the standard model: the quarks, which interact via strong forces, and the *leptons*, including electrons, muons, and neutrinos, which do not. In addition, there is a family of spin-1 bosons, which act as force carriers in the theory, and a spin-0 particle, called the *Higgs boson*, which plays a key role in understanding the origin of elementary particle masses within the theory.

The above particles interact via four forces of nature. In decreasing order of strength, these are the strong interaction, which binds the

quarks together into hadrons; the electromagnetic interaction between the charged leptons and quarks; the weak interaction responsible for β decay; and gravity. Although an understanding of all four forces will ultimately be essential in a complete theory, gravity is so weak that it can be neglected in nuclear and particle physics at presently accessible energies. Because of this, we will often refer in practice to the three forces of nature. The standard model specifies the origin of these three forces. In classical physics the electromagnetic interaction is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of photons, which are members of the family of fundamental spin-1 bosons of the standard model. Photons are referred to as the *gauge bosons*, or ‘force carriers’, of the electromagnetic interaction. The use of the word ‘gauge’ originates from the fact that the electromagnetic interaction possesses a fundamental symmetry called *gauge invariance*. For example, Maxwell’s equations of classical electromagnetism are invariant under a specific transformation of the electromagnetic fields, called a gauge transformation. This property is common to all the three interactions of nature we will be discussing and has profound consequences, but we will not need its details in this book.¹⁴ The weak and strong interactions are also mediated by the exchange of spin-1 gauge bosons. For the weak interaction these are the W^+ , W^- , and Z^0 bosons (again the superscripts denote the electric charges) with masses about 80–90 times the mass of the proton. For the strong interaction, the force carriers are called *gluons*. There are eight gluons, all of which have zero mass and are electrically neutral.

In addition to the elementary particles of the standard model, there are other important particles we will be studying. These are the *hadrons*, the bound states of quarks. Nucleons are examples of hadrons, but there are several hundred more, not including nuclei, most of which are unstable and decay by one of the three interactions. For example, the charged pions π^\pm decay via the weak interaction with a lifetime of about 10^{-8} s, while the neutral pion π^0 decays via the electromagnetic interaction with a lifetime of about 10^{-17} s. The existence of quarks was first inferred from the properties of hadrons, as we have seen, and they remain particularly important because free quarks are unobservable in nature. Hence to deduce properties of quarks we are forced to study hadrons. An analogy would be if we had to deduce the properties of nucleons by exclusively studying the properties of nuclei.

Since nucleons are bound states of quarks and nuclei are bound states of nucleons, the properties of nuclei should in principle be deducible from

¹⁴A brief description of gauge invariance and some of its consequences is given, for the interested reader, in Appendix D.

the properties of quarks and their interactions, that is, from the standard model. Although there has been some progress in this direction, in practice this is still beyond present calculational techniques and often nuclear and particle physics are treated as two almost separate subjects. However, there remain some connections between them and in introductory treatments it is still useful to present both subjects together.

The remaining sections of this chapter are devoted to introducing some of the basic theoretical tools needed to describe the phenomena of both nuclear and particle physics, starting with a key concept in the latter: antiparticles.

1.2 Relativity and antiparticles

Elementary particle physics is also called high-energy physics. One reason for this is that if we wish to produce new particles in a collision between two other particles, then because of the relativistic mass–energy relation $E = mc^2$, energies are needed at least as great as the rest masses of the particles produced. The second reason is that to explore the structure of a particle requires a probe whose wavelength λ is smaller than the structure to be explored. By the de Broglie relation $\lambda = h/p$, this implies that the momentum p of the probing particle, and hence its energy, must be large. For example, to explore the internal structure of the proton using electrons requires wavelengths that are much smaller than the radius of the proton, which is roughly 10^{-15} m. This in turn requires electron energies that are greater than 10^3 times the rest energy of the electron, implying electron velocities very close to the speed of light. Hence any explanation of the phenomena of elementary particle physics must take account of the requirements of the theory of special relativity, in addition to those of quantum theory. There are very few places in particle physics where a nonrelativistic treatment is adequate, whereas the need for a relativistic treatment is much less in nuclear physics.

Constructing a quantum theory that is consistent with special relativity leads to the conclusion that for every charged particle of nature, there must exist an associated particle, called an *antiparticle*, with the same mass as the corresponding particle. This important prediction was first made by Dirac and follows from the solutions of the equation he postulated to describe relativistic electrons.¹⁵ The *Dirac equation* for a particle of mass m and momentum \mathbf{p} moving in free space is of the form

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = H(\mathbf{r}, \hat{\mathbf{p}}) \Psi(\mathbf{r}, t), \quad (1.1)$$

¹⁵Paul Dirac shared the 1933 Nobel Prize in Physics with Erwin Schrödinger. The somewhat cryptic citation stated ‘for the discovery of new productive forms of atomic theory’.

where we use the notation $\mathbf{r} = (x_1, x_2, x_3) = (x, y, z)$, $\hat{\mathbf{p}} = -i\hbar\nabla$ is the usual quantum mechanical momentum operator and the Hamiltonian was postulated by Dirac to be

$$H = c \boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta m c^2. \quad (1.2)$$

The coefficients $\boldsymbol{\alpha}$ and β are determined by the requirement that the solutions of (1.1) are also solutions of the free-particle *Klein-Gordon equation*

$$-\hbar^2 \frac{\partial^2 \Psi(\mathbf{r}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \Psi(\mathbf{r}, t) + m^2 c^4 \Psi(\mathbf{r}, t), \quad (1.3)$$

which follows from making the usual quantum mechanical substitutions $\mathbf{p} \rightarrow -i\hbar\nabla$ and $E \rightarrow i\hbar\partial/\partial t$ in the relativistic mass–energy relation $E^2 = p^2 c^2 + m^2 c^4$. This leads to the conclusion that $\boldsymbol{\alpha}$ and β cannot be ordinary numbers; their simplest forms are 4×4 matrices. Thus the solutions of the Dirac equation are four-component wavefunctions (called *spinors*) with the form¹⁶

$$\Psi(\mathbf{r}, t) = \begin{pmatrix} \psi_1(\mathbf{r}, t) \\ \psi_2(\mathbf{r}, t) \\ \psi_3(\mathbf{r}, t) \\ \psi_4(\mathbf{r}, t) \end{pmatrix}. \quad (1.4)$$

The interpretation of (1.4) is that the four components describe the two spin states of a negatively charged electron with positive energy and the two spin states of a corresponding particle having the same mass, but with negative energy. Two spin states arise because in quantum mechanics the projection in any direction of the spin vector of a spin-1/2 particle can only result in one of the two values $\pm 1/2$, referred to as ‘spin up’ and ‘spin down’, respectively. The two energy solutions arise from the two solutions of the relativistic mass–energy relation $E = \pm(p^2 c^2 + m^2 c^4)^{1/2}$. The negative-energy states can be shown to behave in all respects as positively charged electrons called *positrons*, but with *positive* energy.¹⁷ The positron is referred to as the *antiparticle* of the electron. The discovery of the positron by Anderson in 1933, with all the predicted properties, was a spectacular verification of Dirac’s prediction, as was the much later discovery of the antiproton in 1955.¹⁸

Although Dirac originally made his prediction for electrons, the result holds for all charged particles and is true whether the particle is an elementary particle or a hadron. If we denote a particle by P , then the antiparticle is in general written with a bar over it, i.e. \bar{P} . For example,

¹⁶The details may be found in many quantum mechanics books, e.g. pp. 475–477 of Schiff (1968).

¹⁷See, for example, Chapter 1 of Martin and Shaw (2017).

¹⁸Carl Anderson shared the 1936 Nobel Prize in Physics for the discovery of the positron and Emilio Segrè and Owen Chamberlain were awarded the 1959 prize for their discovery of the antiproton.

the antiparticle of the proton p is the antiproton \bar{p} , with negative electric charge, and associated with every quark, q , is an antiquark, \bar{q} . However, for some very common particles the bar is usually omitted. Thus, for example, in the case of the positron e^+ , the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle. The argument does not extend to neutral particles in general and while some have distinct antiparticles, others do not. For example, the neutron has a non-zero magnetic moment, as we shall see below, and there is a distinct antiparticle, the *antineutron* \bar{n} , which has a magnetic moment equal in magnitude to that of the neutron, but opposite in sign. On the other hand, neither the photon γ nor the neutral pion π^0 has a distinct antiparticle.

Electric charge is just one example of a quantum number that has equal and opposite values for particles and antiparticles. We will meet others later. When brought together, particle–antiparticle pairs, each of mass m , can annihilate, releasing their combined rest energy $2mc^2$ as photons or other particles. There is a symmetry between particles and antiparticles, and it is a convention to call the electron the particle and the positron its antiparticle. This reflects the fact that normal matter contains electrons rather than positrons.

Finally, we note that among the many successful predictions of the Dirac equation is that for magnetic moments. A charged particle with spin necessarily has an intrinsic magnetic moment $\boldsymbol{\mu}$, and it can be shown from the Dirac equation that a point-like spin-1/2 particle of charge q and mass m has a magnetic moment $\boldsymbol{\mu} = (q/m) \mathbf{S}$, where \mathbf{S} is its spin vector.¹⁹ Magnetic moment is a vector, and the value μ tabulated is the z component of $\boldsymbol{\mu}$ when the z component of spin has its maximum value, i.e. $\mu = q\hbar/2m$. This is a test of the elementarity of a spin-1/2 particle and the measured magnetic moment of the electron is compatible with this assumption. However, the experimental values for the proton and neutron are

$$\boldsymbol{\mu}_p = 2.79e \mathbf{S}/m_p \quad \text{and} \quad \boldsymbol{\mu}_n = 1.91e \mathbf{S}/m_n,$$

which do not obey the Dirac prediction, reflecting the fact that and the proton and neutron are not point-like, elementary particles.²⁰

1.3 Space-time symmetries and conservation laws

Symmetries and the invariance properties of the underlying interactions play an important role in physics. Some lead to conservation laws that are universal. Familiar examples are translational invariance, leading to

¹⁹There is a small correction to this predicted value, of order one part in a thousand, which we ignore in this simple account. See, for example, Section 9.6 of Mandl and Shaw (2010).

²⁰The proton magnetic moment was first measured by Otto Stern in 1933 using a molecular beam method that he developed and for this he received the 1943 Nobel Prize in Physics.

the conservation of linear momentum; and rotational invariance, leading to conservation of angular momentum. The latter plays an important role in nuclear and particle physics as it leads to a scheme for the classification of states based, among other quantum numbers, on their spins. This is similar to the scheme used to classify states in atomic physics.²¹ Another very important invariance that we have briefly mentioned is gauge invariance. This fundamental property of all three interactions restricts their forms in a profound way. In its simplest form, it predicts zero masses for all elementary particles. However, there are theoretical solutions to this problem whose experimental verification is described in Section 6.5.

In nuclear and particle physics we need to consider additional symmetries of the interactions and the conservation laws that follow. In the remainder of this section we discuss three space–time symmetries that we will need in later chapters – *parity*, *charge conjugation*, and *time-reversal*.

1.3.1 Parity

Parity was first introduced in the context of atomic physics by Wigner in 1927.²² It refers to the behaviour of a state under a spatial reflection, i.e. $\mathbf{r} \rightarrow -\mathbf{r}$. If we consider a single-particle state, represented for simplicity by a nonrelativistic wavefunction $\Psi(\mathbf{r}, t)$, then under the parity operator \hat{P} ,

$$\hat{P}\Psi(\mathbf{r}, t) \equiv P\Psi(-\mathbf{r}, t). \quad (1.5)$$

Applying the operator again gives

$$\hat{P}^2\Psi(\mathbf{r}, t) = P\hat{P}\Psi(-\mathbf{r}, t) = P^2\Psi(\mathbf{r}, t), \quad (1.6)$$

implying $P = \pm 1$. If the particle is an eigenfunction of linear momentum \mathbf{p} , i.e.

$$\Psi(\mathbf{r}, t) \equiv \Psi_{\mathbf{p}}(\mathbf{r}, t) = \exp[i(\mathbf{p} \cdot \mathbf{r} - Et)/\hbar], \quad (1.7)$$

then

$$\hat{P}\Psi_{\mathbf{p}}(\mathbf{r}, t) = P\Psi_{\mathbf{p}}(-\mathbf{r}, t) = P\Psi_{-\mathbf{p}}(\mathbf{r}, t) \quad (1.8)$$

and so a particle at rest, with $\mathbf{p} = \mathbf{0}$, is an eigenstate of parity. The eigenvalue $P = \pm 1$ for a particle at rest is called the *intrinsic parity*, or just the *parity*, of the particle. Parity is a multiplicative quantum number, and thus for many-particle systems the appropriate generalisation of (1.5) is

$$\hat{P}\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, t) \equiv P_1 P_2 \cdots \Psi(-\mathbf{r}_1, -\mathbf{r}_2, \dots, t),$$

with one intrinsic parity factor P_1, P_2, \dots occurring for each particle present.

²¹These points are explored in more detail in, for example, Chapter 5 of Martin and Shaw (2017).

²²Eugene Wigner shared the 1963 Nobel Prize in Physics, principally for his work on symmetries.